***Calculator−free Solutions***

1. (a) (i) $$ or $×5!$ ✓

 (ii) $××5!$ ✓✓

 (b) (i) $6^{4}×5^{2}×6!$ ✓

 (ii) ‘9’ from first set only: $××6!$ ✓

 ‘9’ from second set only: $××6!$ ✓

 no ‘9’ chosen: $××6!$ ✓

 total $=\left(×+×+×\right)×6!$ ✓ [8]

2. (a) (i) $i^{n+2}=i^{n}×i^{2}=-i×-1=i$ ✓

 (ii) $i^{2n+1}=\left(i^{n}\right)^{2}×i=\left(-i\right)^{2}×i=-i$ ✓✓

 (b) $\frac{1-i}{i+\frac{2}{i}}×\frac{i}{i}=\frac{i-i^{2}}{i^{2}+2}=\frac{1+i}{-1+2}=1+i$ ✓✓

 (c) $\left(1+i\right)^{4}-\left(1-i\right)^{4}$

 $=\left[\left(1+i\right)^{2}+\left(1-i\right)^{2}\right]×\left[\left(1+i\right)^{2}-\left(1-i\right)^{2}\right]$

 $=\left[1+2i-1+1-2i-1\right]×\left[1+2i-1-1+2i+1\right]$

 $=0×4i=0$ ✓✓ [7]

3. (a) $5^{3}=5+1×10×6+1×4×15=5+60+60=125$ ✓

 (b) $ , $ ✓✓

 (c) $n^{3}=n+××+××$ ✓✓ [5]

4. (a) (i) $\left[\begin{matrix}0&-1&1\\1&1&-1\\1&1&0\end{matrix}\right]×\left[\begin{matrix}1&1&0\\-1&-1&1\\0&-1&1\end{matrix}\right]=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$ ✓✓

 A and B are inverses of each other. ✓

 (ii) $\left[\begin{matrix}0&-1&1\\1&1&-1\\1&1&0\end{matrix}\right]×\left[\begin{matrix}x\\y\\z\end{matrix}\right]=\left[\begin{matrix}-6\\8\\3\end{matrix}\right]$ ✓

 $\left[\begin{matrix}x\\y\\z\end{matrix}\right]=\left[\begin{matrix}0&-1&1\\1&1&-1\\1&1&0\end{matrix}\right]^{-1}×\left[\begin{matrix}-6\\8\\3\end{matrix}\right]=\left[\begin{matrix}1&1&0\\-1&-1&1\\0&-1&1\end{matrix}\right]×\left[\begin{matrix}-6\\8\\3\end{matrix}\right]=\left[\begin{matrix}2\\1\\-5\end{matrix}\right]$ ✓✓

 $∴x=2, y=1, z=-5$

4. (b) (i) T1 performs a rotation of 180°



 $∴y=-\left(x-1\right)^{2}$ ✓✓

 (ii) Reflection of the line $y=x$ ✓

 T2$=\left[\begin{matrix}0&1\\1&0\end{matrix}\right]$✓

 (iii) New Area $=\left|T\_{3}\right|×10\frac{2}{3}=6×\frac{32}{3}=64 units^{2}$ ✓✓ [12]

5. (a) (i) $y=-3\cos(\left[4\left(x-\frac{π}{12}\right)\right]=-3\cos(\left[4x-\frac{π}{3}\right]))$

 $∴ A=-3 ω=4 θ=-\frac{π}{3}$ ✓✓✓

 (ii) $y=-3\sin(\left(4x+\frac{π}{6}\right))$ ✓✓

 (b)



✓ Scale factor

 ($y=2$ at $x=\frac{5π}{24}$)

✓ Period of $\frac{π}{2}$

✓ Vertical Asymptote

 at $x=\frac{π}{12}$

 [8]

6. (a) (i) $\sqrt{2}×\sqrt{8}=\sqrt{16}=4$ ✓

 (ii) “If $ab$ is irrational, then both $a$ and $b$ must be irrational” ✓

 (b) A ⇒ B: If the triangle has two equal sides, then it is isosceles,

 and therefore it has two congruent sides.

 $∴$ A ⇒ B is valid and True. ✓

 B ⇒ A: If the triangle has two congruent sides, then it is isosceles,

 and therefore it has two equal sides.

 $∴$ B ⇒ A is valid and True. ✓

 $∴$ A ⇔ B

 (c) $∀x\in P, ∃ y\in P :xy\in Q$ ✓✓ [6]

7. Assume $n$ is even and that $n^{3}$ is odd. ✓

 Let $m\in N:n=2m $ is even. ✓

 $n^{3}=\left(2m\right)^{3}=8m^{3}=2\left(4m^{3}\right)$ ✓

 Since $n^{3}$ cannot be both even and odd simultaneously, then this

 is a contradiction. And therefore $n$ must be odd. ✓ [4]